

MINISTRY OF INDUSTRY AND TRADE
HANOI UNIVERSITY OF INDUSTRY



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**STATIC AND DYNAMIC ANALYSIS OF FUNCTIONALLY
GRADED DOUBLY CURVED MICROSHELL**

Specialized: Mechanical Engineering

Code number: 9 52 01 03

TÓM TẮT LUẬN ÁN TIẾN SĨ

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PUBLICATIONS RELATED TO THE THESIS

1. **Duong Khuat Duc**, Linh Nguyen Tuan, Mai Dao Nhu, Nguyen Thi Hong, Tran Van Ke & Phung Van Minh (2024): A novel isogeometric model for dynamic buckling analysis of doubly curved two-directional functionally graded porous shallow microshells in thermal environments via variable length-scale parameters, *Mechanics Based Design of Structures and Machines* (ISI-Q2).
2. **Duong, K.D.**, Mai, D.N., Minh, P.V. et al. An isogeometric approach to free vibration analysis of bi-directional functionally graded porous doubly-curved shallow microshells with variable length-scale parameters. *Front. Struct. Civ. Eng.* 17, 1871-1894 (2024) (ISI-Q1)
3. **Duong, K.D.**, Linh, N.T., Thuy, T.T.T., Mai, D.N. (2025). Static Bending Behavior of Two-Directional Functionally Graded Porous Microshell. In: Hoang Tien, D., Solanki, V.K., Mahmud, J., Nguyen, T.D.L. (eds) *Proceedings of the 1st International Conference on Sustainability and Emerging Technologies for Smart Manufacturing. SETSM 2024. Proceedings in Technology Transfer*. Springer, Singapore. (Scopus)
4. Nguyễn Tuấn Linh, Đào Như Mai, và **Khuất Đức Dương**. Phân tích Dao động Riêng của tấm Micro Cơ tính biến Thiên Theo Hai phương với lỗ rỗng bằng tiếp cận đẳng hình học. *Tạp Chí Khoa học Và Công nghệ - Đại học Đà Nẵng*, vol 21, số p.h 5, Tháng Năm 2023, tr 39-44, <https://jst-ud.vn/jst-ud/article/view/8404>.
5. **Khuat Duc Duong**, Nguyen Tuan Linh (2025). Analytical Method for Calculating the Extreme Value of Static Bending Deflection of Two-Curvature Microshells. *Mechanics of Solids* (ISI-Q3), <https://doi.org/10.1134/S0025654425602824>.
6. **Khuất Đức Dương**, Đào Như Mai, Nguyễn Tuấn Linh, Trần Thị Thu Thủy. Phân tích ổn định của vỏ micro có lỗ rỗng và cơ tính biến thiên hai phương bằng phương pháp đẳng hình học. Hội nghị khoa học và công nghệ toàn quốc về cơ khí lần thứ VII - NSCME20

INTRODUCTION

Reasons for choosing the topic: Thin-shell structures with two curvatures are increasingly used due to their superior features such as load-bearing capacity, high geometric flexibility, and light weight [1]. When the structure size is reduced to the micro level, size-dependent effects begin to play an important and indispensable role. These effects require the correction of classical mechanical models using advanced theories such as stress gradient theory, couple stress theory, or multi-field models. Based on the above practical and scientific needs, this research topic “*Static and dynamic analysis of functionally graded doubly curved microshell*” is necessary and has high scientific and practical significance.

Research objectives: Specifically, the objectives are as follows: To build a model of a BFGP curved microshell based on first-order and classical theories, stress gradient theory, or modified couple stress theory.

Research subject: doubly curved microshells (rectangular or elliptical projections).

Scope of research: Linear analysis of problems: static bending and free vibration, static stability, dynamic stability, and forced vibration problems. Considering the influence of dimension effects through modified couple stress theory.

Scope of research

Linear analysis of the following problems: static bending and free vibration, static stability, dynamic stability, and forced vibration.

Modeling of functionally graded materials (FGMs) with material properties varying in two directions, using the power-law distribution, while taking into account the effects of porosity.

Investigation of size-dependent effects based on the Modified Couple Stress Theory (MCST).

The material layers are assumed to be perfectly bonded without delamination, and the structures are subjected to small loads, such that geometrically nonlinear deformation effects are neglected.

Research methodology:

- Theory: Construct a mathematical model based on first-order shear strain theory and classical shear strain theory combined with improved mass stress pair theory (MCST).

- Method: Use the isogeometric analysis method (IGA).

Scientific and practical significance:

- Scientifically: The thesis contributes to the refinement of the theory of analyzing bi-directional functionally graded porous microshells.
- Practically: The research and calculation results can be applied in the design and optimization of technical components and parts with micro-shell structures.

Thesis Structure: The thesis is presented in four chapters, in addition to the introduction, conclusion, recommendations, and bibliography: Chapter 1: Overview of research on functionally graded doubly curved microshells. Chapter 2: Construction of the mechanical behavior problems of functionally graded doubly curved microshells using the isogeometric analysis method. Chapter 3: Evaluation of the influence of parameters on the static response of functionally graded doubly curved microshells. Chapter 4: Evaluation of the influence of parameters on the dynamic response of functionally graded doubly curved microshells.

CHAPTER 1: OVERVIEW OF RESEARCH ON FUNCTIONALLY GRADED DOUBLY CURVED MICROSHELLS

1.1. Overview of the research

1.1.1 Research on doubly-curved microshells

From the research studies analyzed above, it can be seen that most studies focus on: Homogeneous shells or multilayer composites. Size effects when shells are scaled down to the micro/nano level are not considered. Only one-dimensional functionally graded materials (1D-FGM) are considered.

1.1.2 Research on FGM materials

Figure 1.21 describes the number of scientific publications related to the keyword "FGM material" in the last 39 years. Data was compiled from the Google Scholar academic platform (as of August 2024)[3].

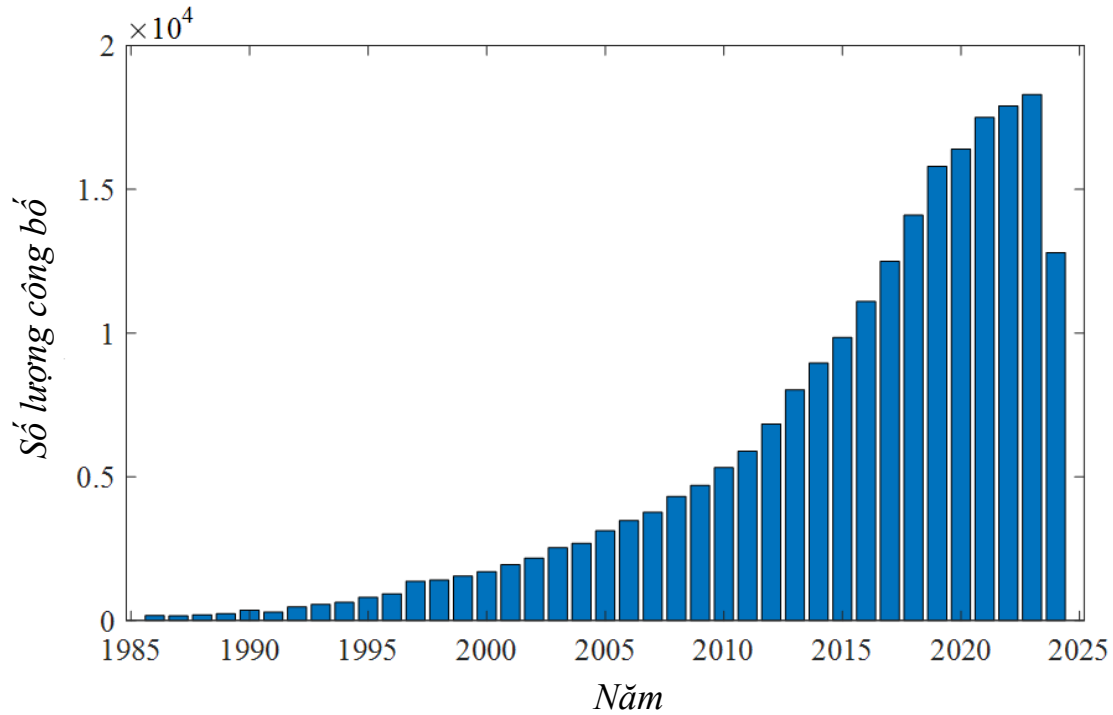


Figure 0.1. The number of scientific publications related to the keyword "FGM material" in the last 39 years

1.1.3 Study of the size dependent

For micro/nano structures, classical theory is no longer accurate enough because it ignores size effects, interparticle interactions, and stress gradients. Several advanced theories exist, such as: Classical Couple Stress Theory (CST); Modified Couple Stress Theory (MCST); Stress Gradient Theory (SGT); Strain Gradient Theory (SGT); or Nonlocal Elasticity.

1.1.4 Optimization Study of FGM Structures

Through analysis, we can see that several research directions have addressed optimizing the FGM distribution to: Reduce structural mass, increase natural vibration frequencies, and reduce stress concentration.

1.1.5 Orientation for applying the research results of the thesis

The research findings of this thesis not only contribute to the refinement of microstructural analysis theory but also have the potential for direct application in key areas of mechanical engineering.

1.1.6 Conclusion of Chapter 1

Chapter 1 presented an overview of the scientific basis and research status, both domestically and internationally, related to two-curvature

shell structures, FGM materials, theories considering size-dependent effects, and approaches to structural optimization.

Regarding the research object: Most existing studies focus on simple beams, plates, or shells (such as cylindrical and spherical shells) at conventional macro-scale dimensions. Comprehensive investigations of doubly curved shells at the micro-scale remain relatively limited.

Research orientation of the dissertation: This dissertation aims to address the static, dynamic, and structural optimization problems of micro-scale doubly curved shells made of two-directional functionally graded materials (2D-FGMs), while accounting for size-dependent effects through advanced theoretical models. The research employs the isogeometric analysis (IGA) method in combination with modern optimization algorithms. The expected results will contribute to improving the theoretical modeling framework and enhancing the accuracy of predicting the mechanical behavior of such structures, while also expanding their potential applications in the design and fabrication of advanced micro-scale engineering systems.

Chapter 2: construction of the mechanical behavior problems of functionally graded doubly curved microshells using the isogeometric analysis method

2.1. Model of the doubly curved micro-shell and assumptions

Consider a doubly curved microshell with a rectangular planform, supported on a Pasternak elastic foundation with geometric parameters as shown in Figure 2.1.

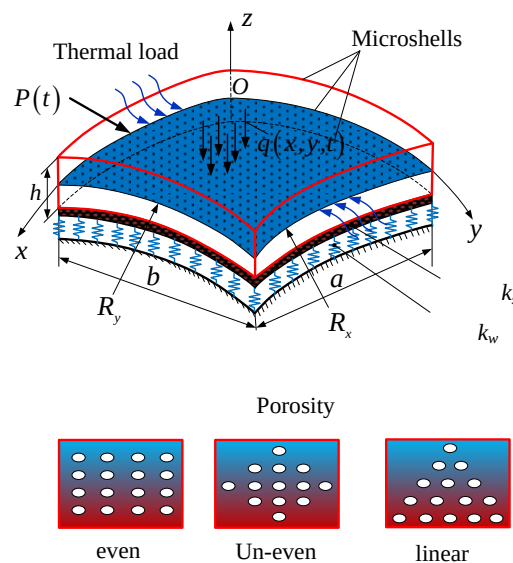


Figure 0.1. Model of microshell with a rectangular planform

For a doubly curved microshell with an elliptical planform with radii R_1, R_2 respectively, it is described as shown in Figure 2.2 below:

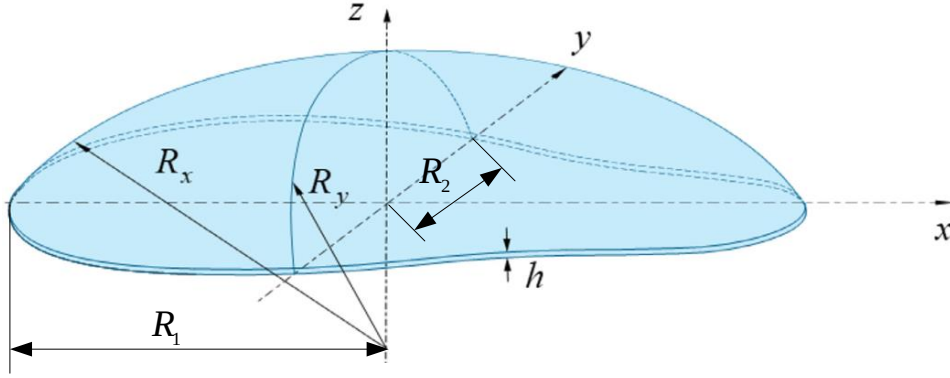


Figure 0.2. Model of microshell with an elliptical planform

2.2. Material model

- For microshell with a rectangular planform:

$$E(x, z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{x}{a} \right)^{\alpha_x} - \vartheta(E_c + E_m) \quad (2.1a)$$

$$\rho(x, z) = \rho_m + (\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{x}{a} \right)^{\alpha_x} - \vartheta(\rho_c + \rho_m) \quad (2.1b)$$

$$\bar{\alpha}(x, z) = \bar{\alpha}_m + (\bar{\alpha}_c - \bar{\alpha}_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{x}{a} \right)^{\alpha_x} - \vartheta(\bar{\alpha}_c + \bar{\alpha}_m) \quad (2.1c)$$

$$v(x, z) = v_m + (v_c - v_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{x}{a} \right)^{\alpha_x} - \vartheta(v_c + v_m) \quad (2.1d)$$

- For microshell with an elliptical planform:

$$E(x, z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{|x|}{R_1} \right)^{\alpha_x} - \vartheta(E_c + E_m) \quad (2.2a)$$

$$\rho(x, z) = \rho_m + (\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{|x|}{R_1} \right)^{\alpha_x} - \vartheta(\rho_c + \rho_m) \quad (2.1b)$$

$$\bar{\alpha}(x, z) = \bar{\alpha}_m + (\bar{\alpha}_c - \bar{\alpha}_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{|x|}{R_1} \right)^{\alpha_x} - \vartheta(\bar{\alpha}_c + \bar{\alpha}_m) \quad (2.1c)$$

$$v(x, z) = v_m + (v_c - v_m) \left(\frac{1}{2} + \frac{z}{h} \right)^{\alpha_z} \left(\frac{|x|}{R_1} \right)^{\alpha_x} - \vartheta(v_c + v_m) \quad (2.1d)$$

Table 0.1. Some material properties and parameters [4]

material	properties				
	E (GPa)	ρ (kg/m ³)	ν	α (K ⁻¹)	β (%H ₂ O) ⁻¹
<i>Al</i>	70	2707	0.3	23x10 ⁻⁶	0.44
<i>Al₂O₃</i>	380	3800	0.3	7x10 ⁻⁶	0.001

2.3. Displacement and Strain Fields

Based on Kirchhoff-Love's classical shear deformation theory [5], we have the displacement field:

$$\begin{cases} u_x = \left(1 + \frac{z}{R_x} \right) u_0(x, y, t) - zw_{0,x}(x, y, t) \\ u_y = \left(1 + \frac{z}{R_y} \right) v_0(x, y, t) - zw_{0,y}(x, y, t) \\ u_z = w_0(x, y, t) \end{cases} \quad (2.3)$$

The strain field can be rewritten as a vector as follows:

$$\begin{aligned} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} &= [1 \quad z \quad z^2] \boldsymbol{\varepsilon}^b; \begin{Bmatrix} \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} = [1 \quad z] \boldsymbol{\varepsilon}^s; \boldsymbol{\varepsilon}^b = \\ &\begin{Bmatrix} \boldsymbol{\varepsilon}_1^b \\ \boldsymbol{\varepsilon}_2^b \\ \boldsymbol{\varepsilon}_3^b \end{Bmatrix}; \boldsymbol{\varepsilon}^s = \begin{Bmatrix} \boldsymbol{\varepsilon}_1^s \\ \boldsymbol{\varepsilon}_2^s \end{Bmatrix}; \end{aligned} \quad (2.4)$$

2.4. Relationship between stress and strain

2.4.1. Modified couple stress theory.

Based on the MCST, the stress-strain relationship of a microshell is described as follows [6]:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\kappa \varepsilon_{ij} - \Delta T \bar{\alpha}(x, z) \delta_{ij} \quad (2.5a)$$

$$\psi_{ij} = 2\kappa l \theta_{ij} \quad (2.5b)$$

2.4.2. Expanded form of the stress-strain relationship

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{Bmatrix} - \Delta T \bar{\alpha}(x, z) \begin{Bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} \mathbf{Q}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^b \\ \boldsymbol{\varepsilon}^s \end{Bmatrix} - \Delta T \bar{\alpha}(x, z) \boldsymbol{\varepsilon}^{th}; \quad (2.6)$$

2.5. Energy components and weak forms

The strain energy of the of the micro-shell $\delta\mathcal{U}$ is determined as follows:

$$\begin{aligned} \delta\mathcal{U} &= \int_{S_e} \int_{-h/2}^{h/2} \{ \sigma_{ij} \delta \varepsilon_{ij} + \psi_{ij} \delta \theta_{ij} \} dz dx dy \\ &= \int_{S_e} (\hat{\boldsymbol{\sigma}} \delta \hat{\boldsymbol{\varepsilon}} + \hat{\boldsymbol{\psi}} \delta \hat{\boldsymbol{\theta}}) dx dy \end{aligned} \quad (2.7)$$

2.6. Solution Method

2.6.1. IGA method

The IGA control grid model is described in Figure 2.3:

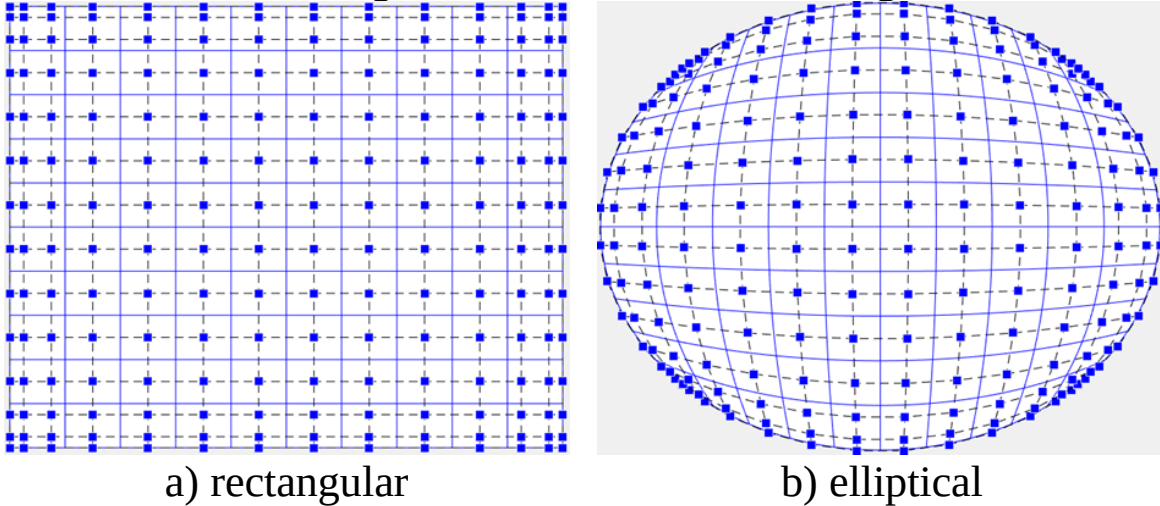


Figure 0.3. The shell's control point model is an isogeometric method.

The differential equation describing the oscillation state of the shell can be expressed in a concise form as follows [7]:

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{C} \dot{\mathbf{q}} + \mathbf{K}_L \bar{\mathbf{q}} = \mathbf{F}(t) \quad (2.8)$$

2.6.2. Boundary Conditions

The boundary conditions used in this thesis are: CCCC, SSSS.

2.6.3. Basic Problems

a. Static bending problem

$$K\bar{q} = F \quad (2.9)$$

b. The problem of free oscillation

$$[K - \omega_{mn}^2 M]\bar{q} = 0 \quad (2.10)$$

c. Dynamic stability problem

$$M\ddot{q} + [K + K_f + K_T - P_{cr}(\beta + \psi \cos(\omega t))K_G]\bar{q} = 0 \quad (2.11)$$

d. Forced oscillation problem

$$M\ddot{q} + C\dot{q} + K_L q = F(t) \quad (2.12)$$

2.7. Kết luận chương 2

This chapter has established the fundamental and important structural behavior problems, including: the static bending problem, the natural vibration problem, the dynamic stability problem, and the dynamics problem.

Some Notable Novel Contributions of the Dissertation in Chapter 2

Modeling of micro-scale doubly curved shells made of two-directional functionally graded materials (2D-FGMs) based on the Modified Couple Stress Theory (MCST).

Considering the length-scale parameter as a variable quantity, similar to other mechanical properties of the material.

The Isogeometric Analysis (IGA) method is combined with the Botion and Newmark methods to formulate the governing equations for dynamic stability, free vibration, and forced vibration problems.

Formulation of fundamental problems for micro-scale doubly curved shells made of two-directional functionally graded materials, together with corresponding flowcharts for solving these problems, providing a basis for implementing computational programs using MATLAB 2018a

Chapter 3: Evaluation of the influence of parameters on the static response of functionally graded doubly curved microshells

3.1. Setting input parameters

4.1.1. Assessing the reliability of the method

Table 0.1. Compare the natural frequency Ω_1^* of FG microplate

		$\alpha_z = 0$		$\alpha_z = 1$		$\alpha_z = 10$	
a/h	l/h	[8]	Thesis	[8]	Thesis	[8]	Thesis
20	0.2	6.4027	6.4424	4.9568	4.9834	4.0323	4.0657
	0.4	7.6708	7.7081	6.0756	6.1016	4.7488	4.7755
	0.8	11.4108	11.4478	9.2887	9.3171	6.9013	6.9216
	1	13.5545	13.5934	11.1042	11.1352	8.1494	8.1691
100	0.2	6.4534	6.4551	4.9922	4.9940	4.0725	4.0751
	0.4	7.7217	7.7233	6.1126	6.1146	4.7840	4.7866
	0.8	11.4689	11.4704	9.3344	9.3370	6.9345	6.9375
	1	13.6186	13.6202	11.1560	11.1589	8.1846	8.1880

3.2. Assessing the impact of input parameters.

4.1.2. Influence of the material's volume coefficient

Table 0.2. The dimensionless first frequency result $\bar{\Omega}_1$ of the microshell with rectangular planform (CCCC)

		$\alpha_x = \alpha_z = \alpha$						
Shell	Porous	l_c/h	0	0.5	1	2	5	10
SP shell	E	0	4.6354	2.3974	2.0116	1.7929	1.6866	1.6681
		0.5	5.9442	2.7776	2.2593	1.9691	1.8257	1.7987
		1	8.7633	3.6768	2.8673	2.4173	2.1890	2.1420
	U	0	4.4274	2.4030	2.0319	1.8186	1.7137	1.6952
		0.5	5.7732	2.8257	2.3149	2.0256	1.8808	1.8527
		1	8.6273	3.8091	2.9982	2.5428	2.3086	2.2588
	CY shell	0	4.3139	2.2175	1.8575	1.6535	1.5535	1.5356
		0.5	5.6960	2.6239	2.1233	1.8430	1.7032	1.6762
		1	8.5972	3.5620	2.7615	2.3156	2.0876	2.0398
	U	0	4.1199	2.2231	1.8765	1.6769	1.5778	1.5599

SAD shell	E	0.5	5.5398	2.6741	2.1797	1.8993	1.7575	1.7294
		1	8.4732	3.6979	2.8950	2.4432	2.2090	2.1584
		0	4.0688	2.0787	1.7388	1.5468	1.4511	1.4332
		0.5	5.5128	2.5089	2.0209	1.7479	1.6102	1.5827
		1	8.4777	3.4793	2.6843	2.2408	2.0123	1.9636
		0	3.8854	2.0838	1.7564	1.5685	1.4736	1.4554
	U	0.5	5.3678	2.5606	2.0778	1.8043	1.6644	1.6356
		1	8.3625	3.6177	2.8198	2.3702	2.1355	2.0839
		0	3.6136	1.8195	1.5156	1.3443	1.2572	1.2398
		0.5	5.1857	2.2995	1.8334	1.5721	1.4380	1.4099
		1	8.2696	3.3322	2.5470	2.1072	1.8775	1.8271
		0	3.4499	1.8243	1.5311	1.3625	1.2754	1.2578
plate	E	0.5	5.0612	2.3549	1.8920	1.6289	1.4918	1.4624
		1	8.1701	3.4758	2.6866	2.2401	2.0041	1.9508
		0	3.6136	1.8195	1.5156	1.3443	1.2572	1.2398
	U	0.5	5.0612	2.3549	1.8920	1.6289	1.4918	1.4624
		1	8.1701	3.4758	2.6866	2.2401	2.0041	1.9508
		0	3.6136	1.8195	1.5156	1.3443	1.2572	1.2398

4.1.3. Influence of length-scale parameters

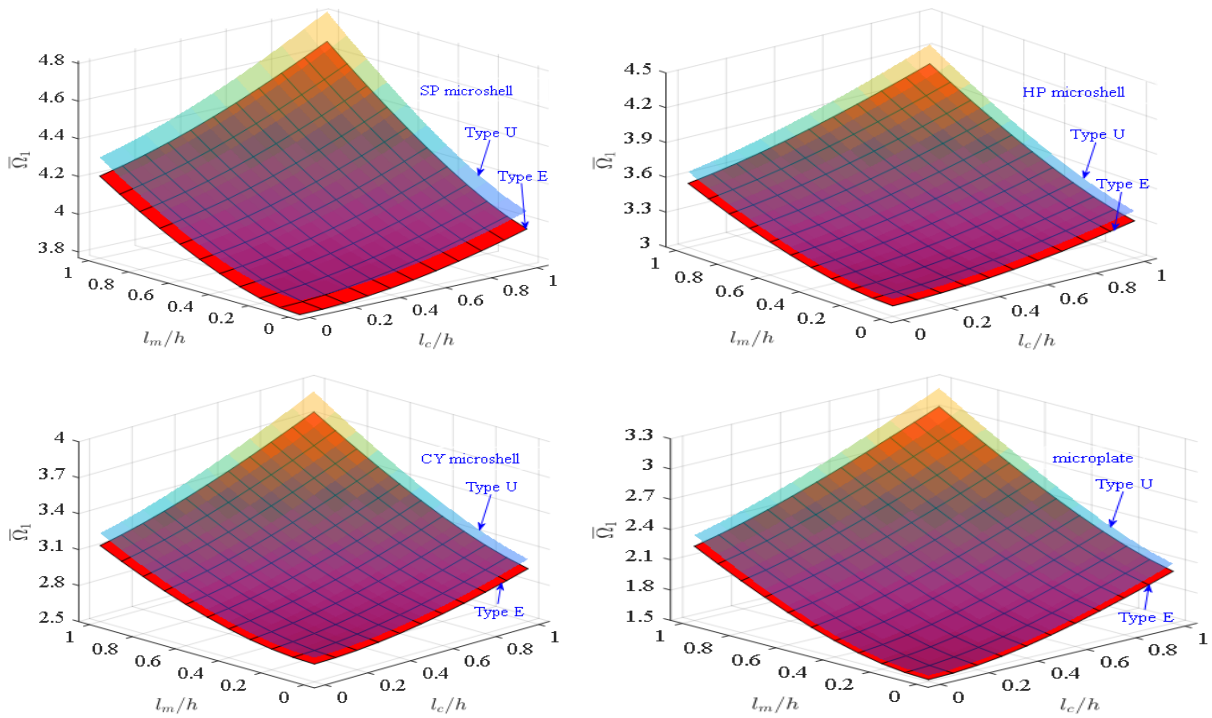


Figure 0.1. Impact of length-scale parameters l_c, l_m on frequency $\bar{Q}_1$ of the microshell with rectangular planform (SSSS, $\Delta T = 0$)

4.1.4. Influence of radius of curvature

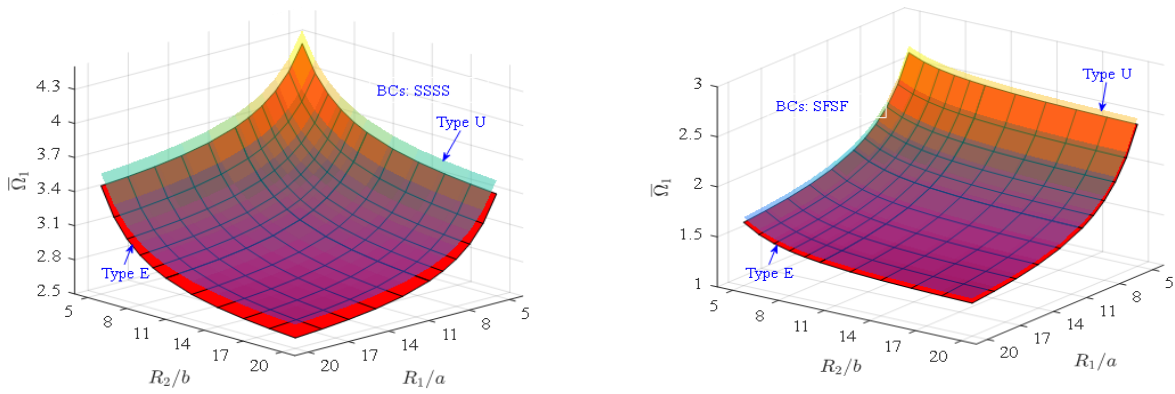


Figure 0.2. Impact of radius of curvature l_c, l_m on frequency $\bar{\Omega}_1$ of the microshell with rectangular planform ($\Delta T = 0$)

4.1.5. Some static bending results

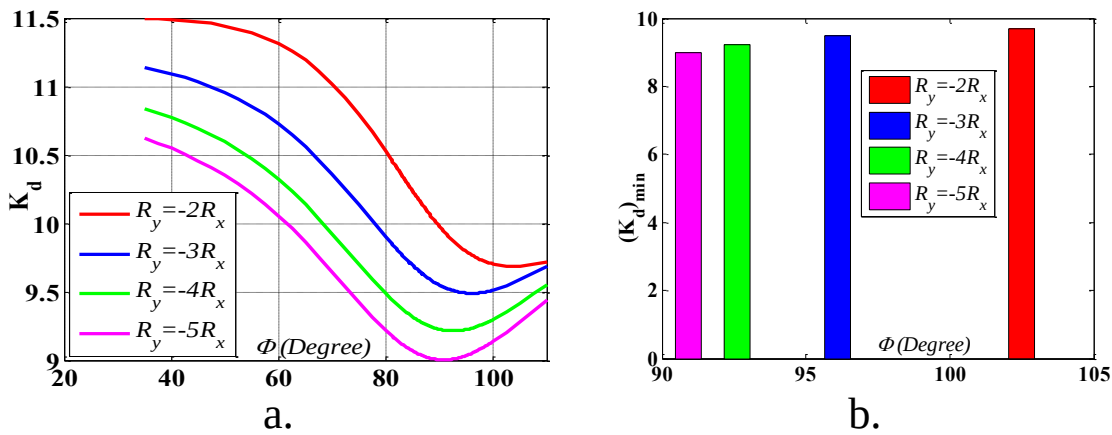


Figure 0.3. The deflection of the midpoint of the micro saddle shell with different values of the ratio R_y/R_x

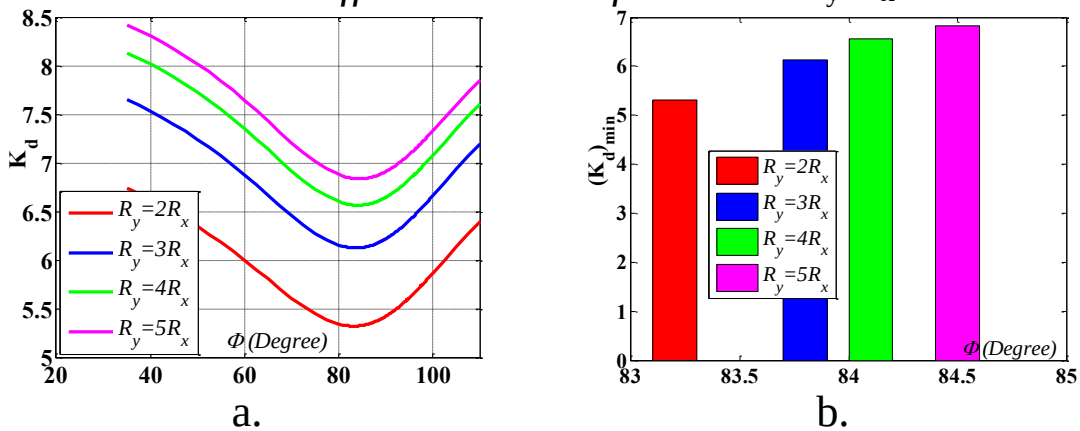
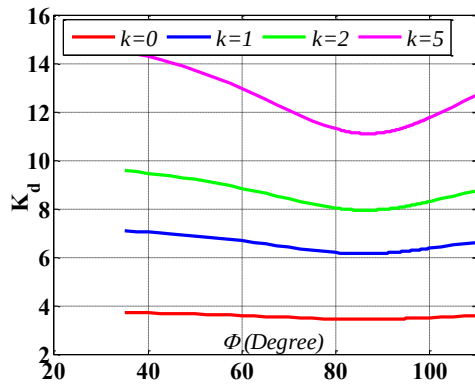
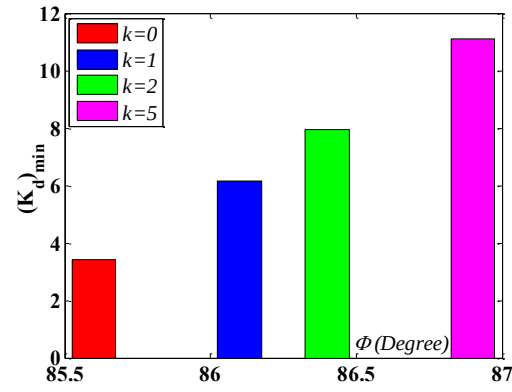


Figure 0.4. The deflection of the midpoint of the micro-sphere shell with different values of the ratio R_y/R_x

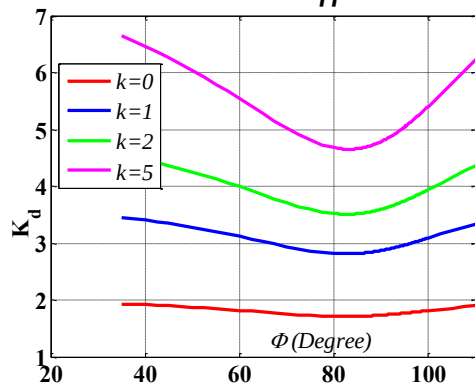


a.

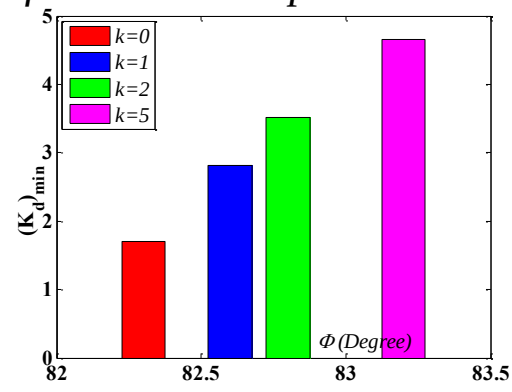


b.

Figure 0.5. The deflection of the midpoint of the micro-cylindrical shell with different values of the volume exponent k

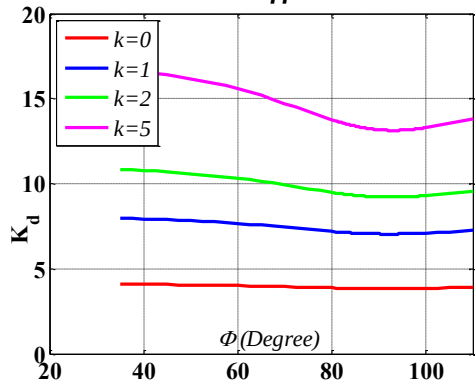


a.

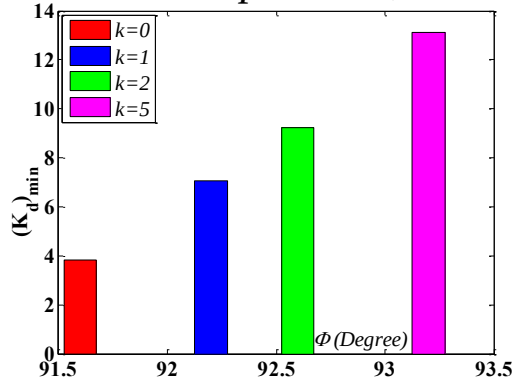


b.

Figure 0.6. The deflection of the midpoint of the micro-sphere shell with different values of the volume exponent k ,



a.



b.

Figure 0.7. The deflection of the midpoint of the micro saddle shell with different values of the volume exponent k

3.3 Conclusion of Chapter 3

In Chapter 3, the thesis presented the algorithmic flowchart and used Matlab 2020b software to develop a program for calculating static bending, natural vibrations, and static stability of a micro-shell with two curvatures supported on a Pasternak elastic foundation.

As the volume fraction of the material increases, the proportion of the softer metallic constituent in the shell structure increases. Consequently, the structural stiffness decreases, leading to reductions in both the natural frequencies and the static buckling load of the micro-shell.

For micro-scale structures, the length-scale parameter has a stiffening effect because it increases the strain energy of the structure. Consequently, both the natural frequencies and the buckling loads increase.

The elastic foundation has an interactive effect and contributes to increasing the structural stiffness, whereas the presence of porosity reduces the mechanical properties and mass of the structure. However, due to the interaction between stiffness and mass, the natural frequencies may either increase or decrease. Nevertheless, the deflection and buckling load inevitably decrease.

Under thermal environmental conditions, the temperature difference between the top and bottom surfaces induces a thermal stiffness matrix, which reduces the stiffness of the micro-structure. Consequently, the corresponding structural response quantities decrease.

Chapter 4: Evaluation of the influence of parameters on the dynamic response of functionally graded doubly curved microshells

4.2. Dynamic Load Model

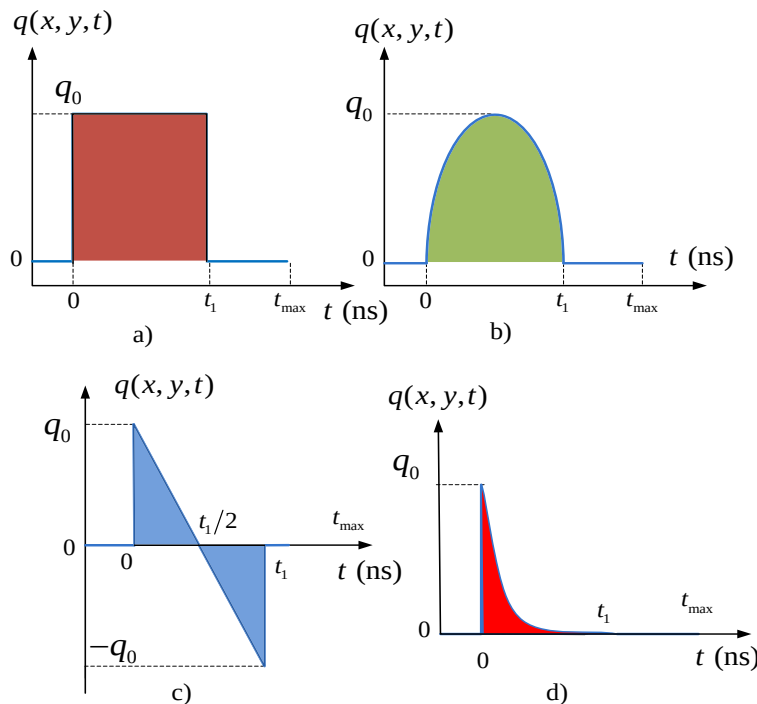


Figure 0.1. Dynamic load models: a) rectangular impulse load (b) sine impulse load (c) triangular impulse load and (d) explosive impulse load (shock wave)

4.3. The problem of evaluating the accuracy of the method

4.3.1. The problem of dynamic stability

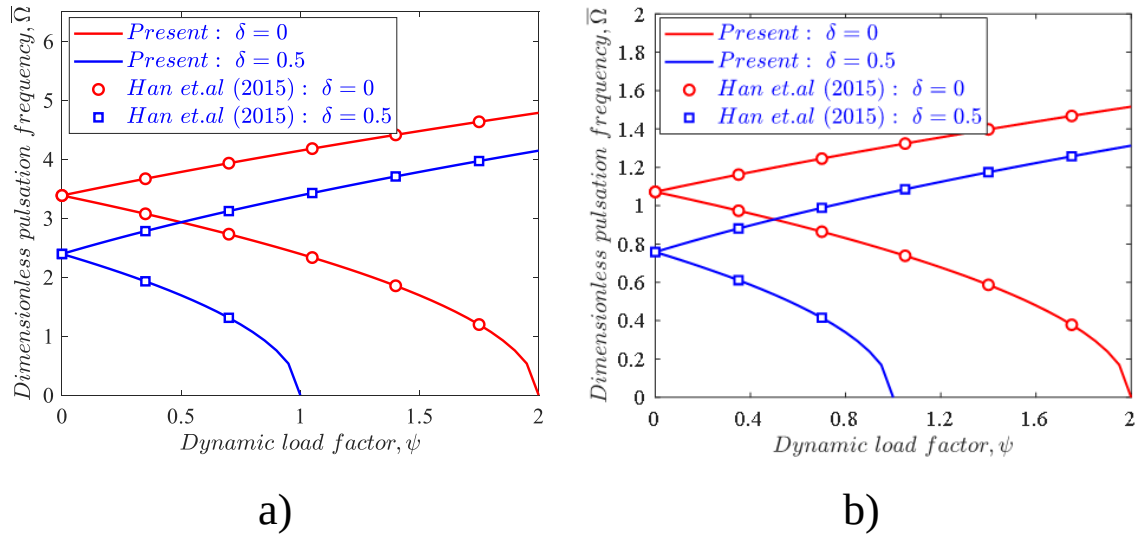


Figure 0.2. The influence of the static force coefficient β on the instability region of the S-FGM plate

4.3.2. Verification problem for the dynamics problem

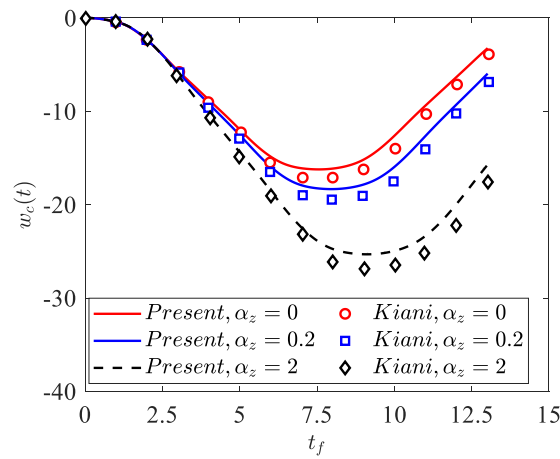


Figure 0.3. Responding to the deflection of a square figure under the effect of a rectangular load over time.

4.4. Results of the dynamic stability problem

4.4.1. Influence of the length scale coefficient

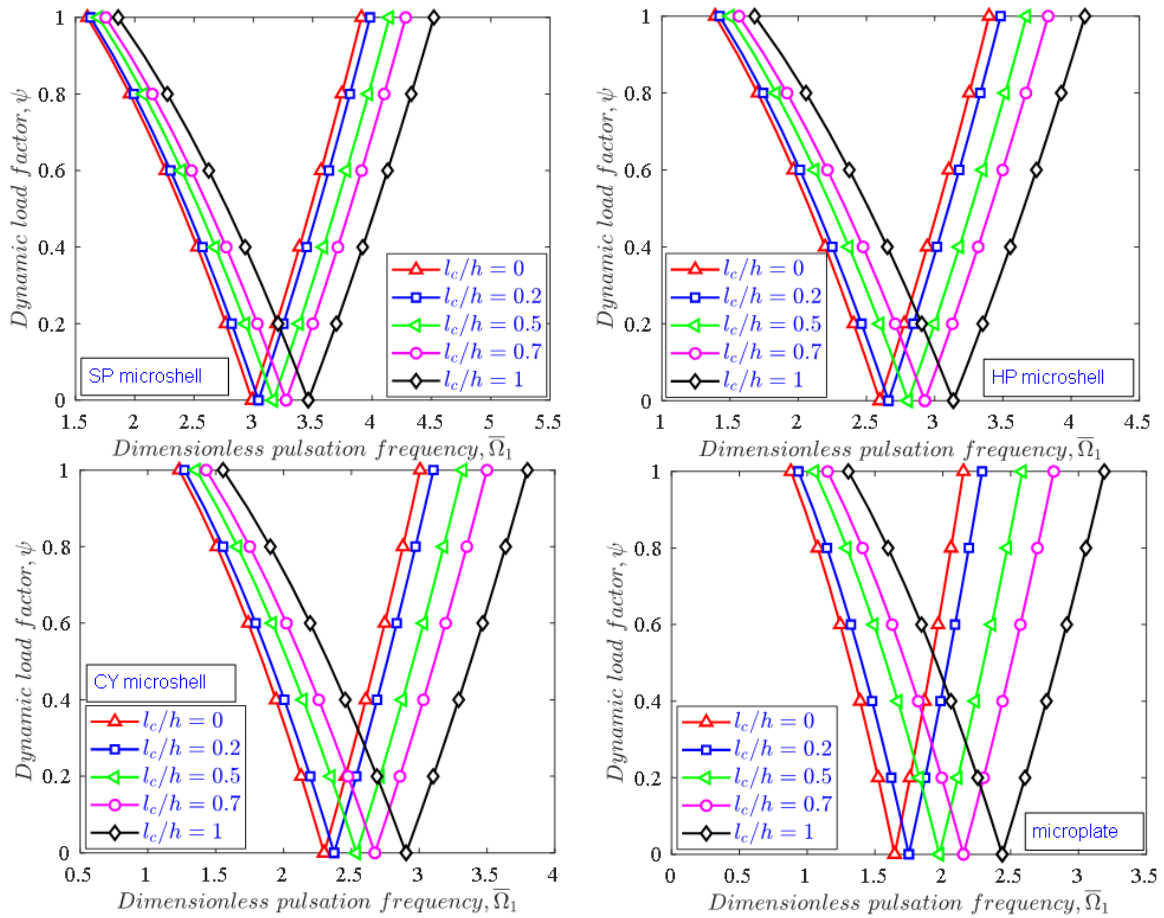


Figure 0.4. The influence of the material length ratio l_c/h on the instability region of the cylindrical microshell with rectangular planform (SSSS, Type E)

4.4.2. Ảnh hưởng của sự chênh lệch nhiệt độ

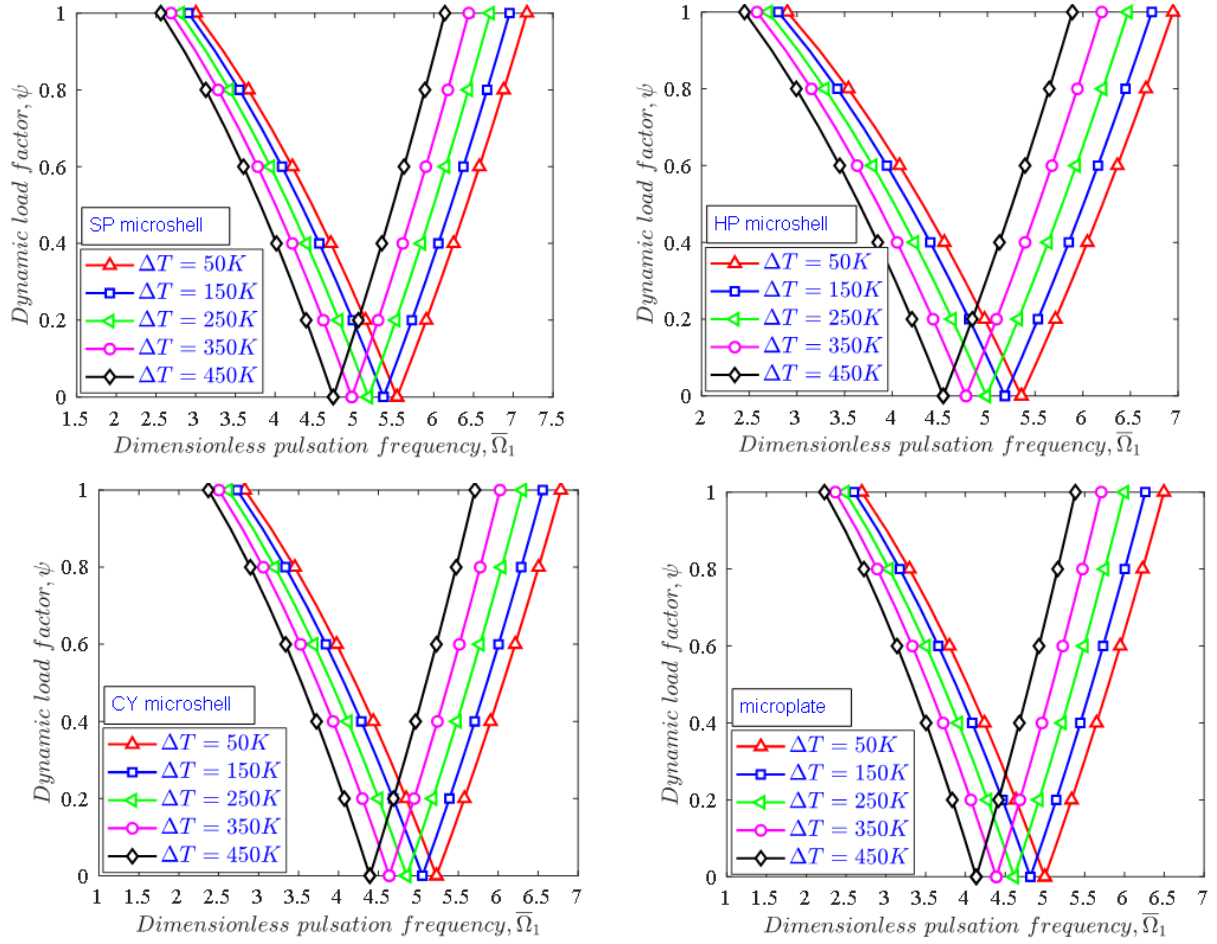
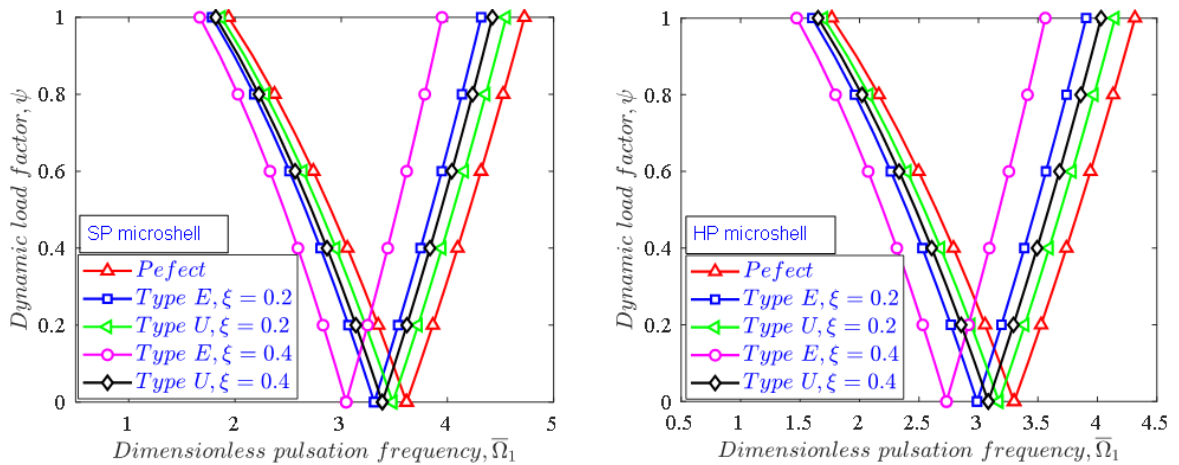


Figure 0.5. The effect of temperature difference $\Delta T(K)$ on the instability region of the micro-cylindrical shell with rectangular planform (CCCC, Type E)

4.4.3. Influence of porosity coefficient and static stability coefficient β



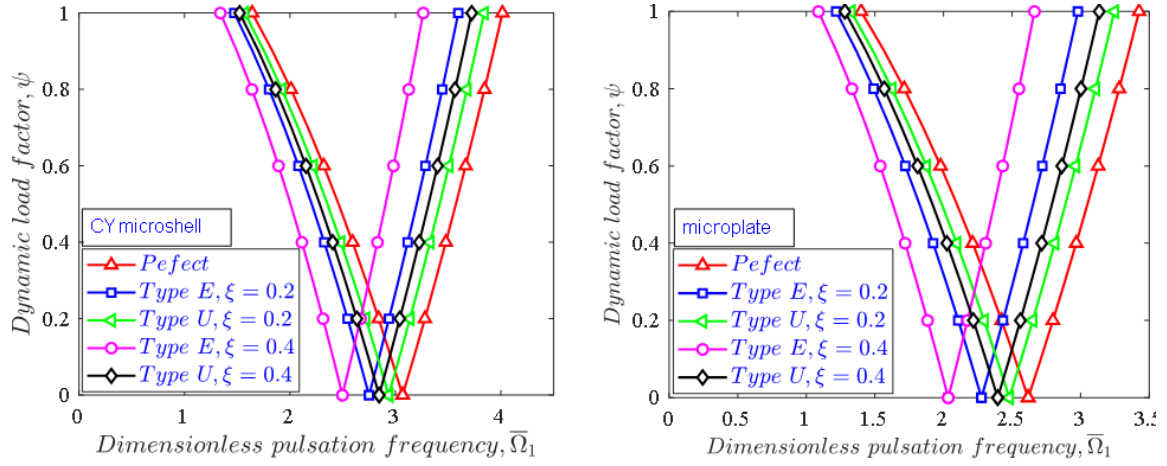


Figure 0.6. Impact of porosity coefficient ξ on the instability region of the micro-cylindrical shell with rectangular (SSSS)

4.4.4. Influence of the material's volume coefficient

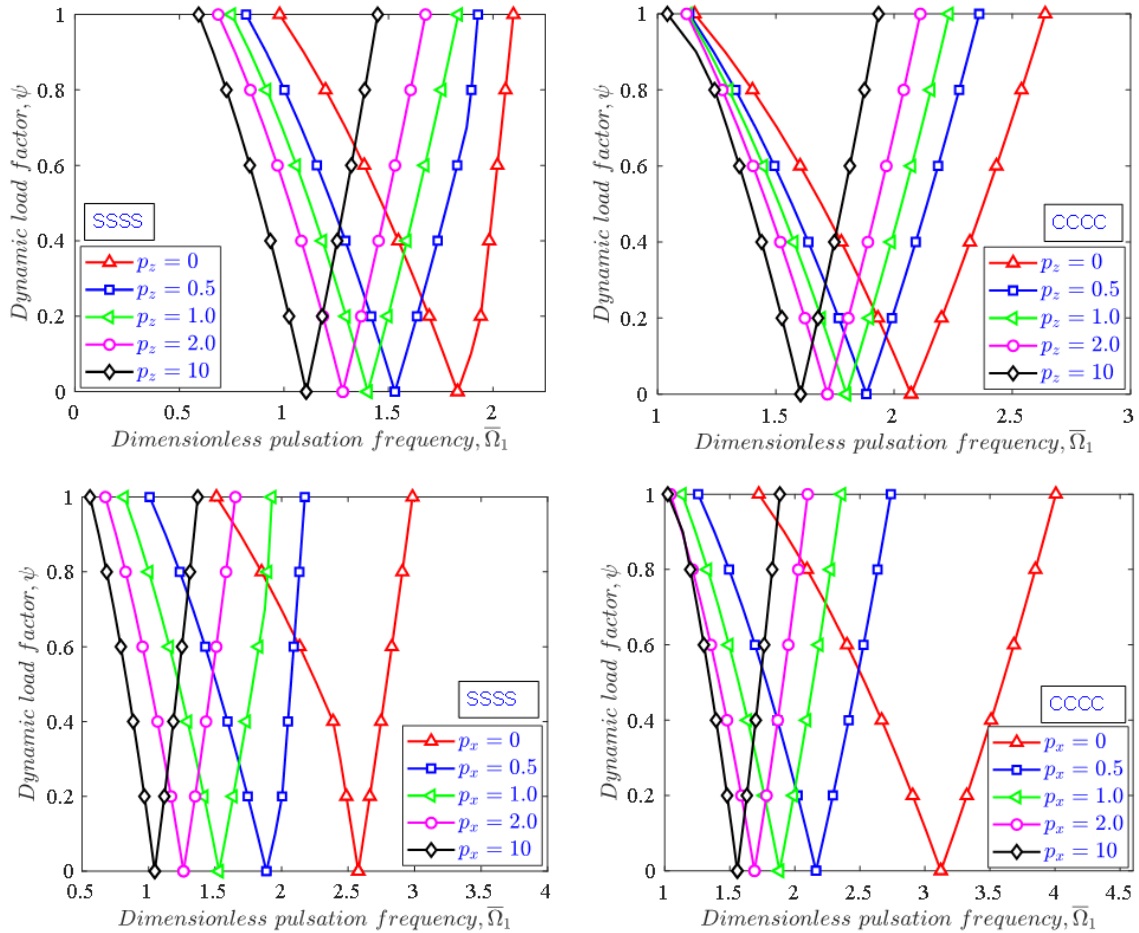


Figure 0.7. Influence of the material's volume coefficient (p_x, p_z) on the instability region of the micro-spherical shell with rectangular

4.5. Results of forced oscillation response

4.5.1. Influence of material volume control coefficient

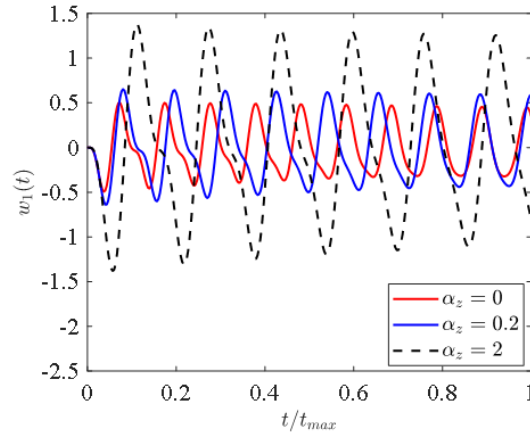


Figure 0.8. The influence of the pz coefficient on the deflection response of the spherical shell under sinusoidal load (SSSS)

4.5.2. Influence of the length scale factor

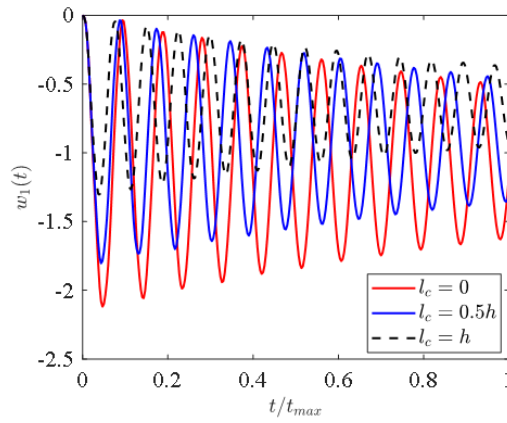


Figure 0.9. The influence of the length scale coefficient on the deflection response of cylindrical shells under rectangular loading. (SSSS)

4.5.3. Influence of porosity coefficient

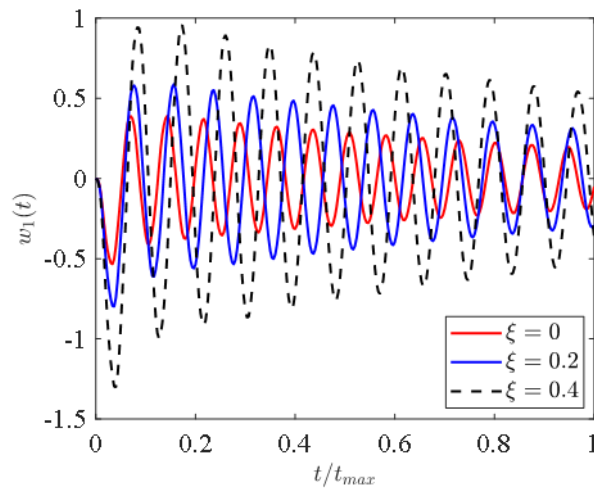


Figure 0.10. The influence of porosity coefficient on the deflection response of saddle shells under explosive load. (SSSS)

4.6 Orientation for applying the research results of the project.

The research results of this thesis have wide applications in many areas of modern mechanical engineering, especially in the context of increasingly smaller mechanical systems and components, requiring high precision and operating in environments with complex loads. First, the eigenvectors, dynamic stability, and forced response models developed in this thesis can be directly applied in the design of sensors and actuators for microelectromechanical systems (MEMS/NEMS). By accurately describing the influence of size effects using modified couple stress theory, the thesis models allow for the prediction of the oscillation behavior of sensor membranes, resonant disks, and microstructured elements that are highly sensitive to model errors at the microscale. This is particularly important in optimizing eigenfrequency, improving sensitivity, and reducing vibration noise for micro-mechanical components.

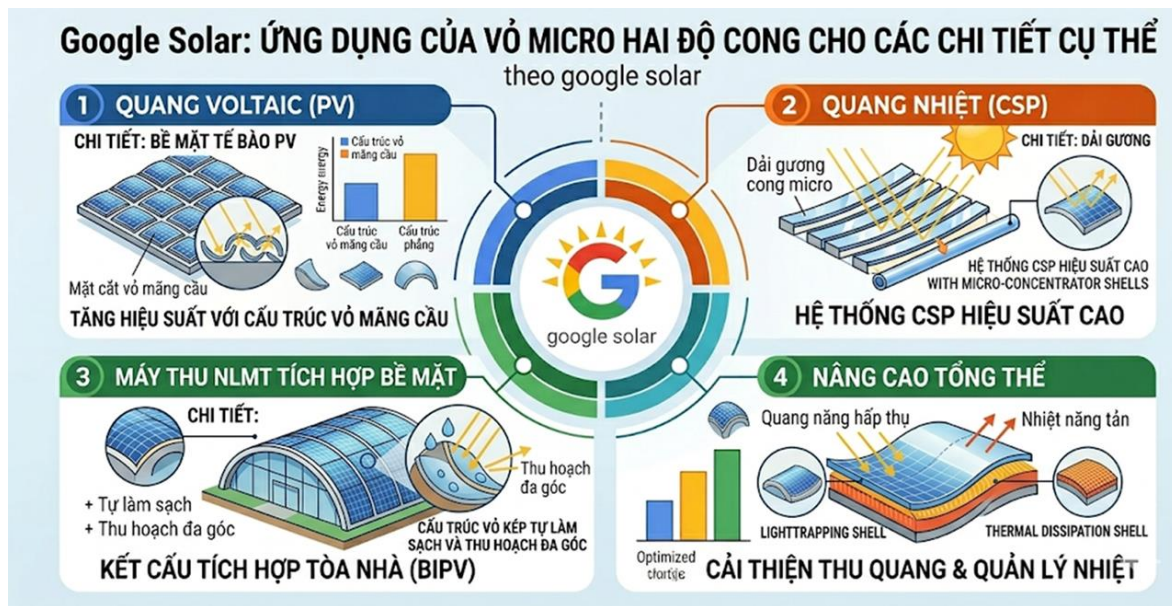


Figure 4.18 Application orientation for BFGP microshell.

Currently, the problem of providing energy to maintain the operation of space exploration devices is a major challenge for scientists. Space exploration drones are becoming increasingly sophisticated, but they cannot function without the necessary energy. In this doctoral thesis, a model of a two-curved micro-shell was developed, analyzing the influence of the material's pore size coefficient, the size effect of the two-curved micro-shell, and performing geometric optimization calculations to achieve extreme values in the static bending problem. The research results of this thesis are directly applied to problems such as designing photovoltaic cell surfaces to increase the structural efficiency of the shell; photothermal devices to increase heat absorption rates; integrated dome-structured solar energy collectors; and devices to improve light collection and heat management, as shown in Figure 4.18, aiming to optimize the efficiency and photothermal energy collection capabilities for future micro-cell systems.

4.7 Conclusion of Chapter 4

This chapter presents the results of dynamic stability and forced vibration analysis of a two-curvature microshell under the action of different dynamic load conditions applied to an elastic foundation, taking into account the small size effect.

Lower-order vibration modes primarily represent the participation of the overall mass and stiffness of the structure, thereby governing its fundamental response under dynamic loading.

Higher-order vibration modes reflect local deformation and consequently increase stress concentrations at locations where the curvature changes.

Opposite curvatures generate torsional and transverse shear effects, resulting in a significant reduction in the natural frequencies compared with those predicted by simplified theoretical models.

Curvatures in the same direction enhance the bending stiffness, allowing the structure to redistribute internal forces and limit the amplitude of resonant vibrations.

The combination of geometric parameters alters the spatial distribution of mass and stiffness, directly affecting the resonance frequency range and the damping characteristics of the overall system.

Comparison between static and dynamic analyses: While static analysis only describes the distribution of deflection and internal forces under equilibrium conditions, dynamic analysis demonstrates that geometric parameters, such as the radius of curvature and thickness-to-length ratio, have a strong influence on the resonance range and potential failure modes—effects that cannot be predicted from static analysis alone.

CONCLUSIONS AND RECOMMENDATIONS

1. New contributions of the thesis

This thesis has modeled and analyzed the static bending responses, natural vibrations, static and dynamic stability, and forced vibrations of a two-curved microshell made of a material with variable mechanical properties in two directions supported on an elastic foundation. The following are some of the main contributions of the thesis:

- A model of the two-curved microshell was constructed from a material with variable mechanical properties along both the length and thickness of the shell, while also considering the influence of small size-dependent effects.

- The length-to-weight ratio coefficient varies like other mechanical properties of the material.

- The thesis presents classical shear strain theory, first-order shear strain theory combined with improved stress-pair theory to fully analyze the mechanical behavior of the microshell structure, such as static bending, natural vibrations, stability, and forced vibrations.

- The basic problems of the microshell structure and their corresponding block diagrams were developed for programming calculations in Matlab software.

- Initially, using optimization knowledge to find suitable sets of values helps to shorten calculation time in practical studies.

- The research results serve as a reference in the calculation and design of nanostructures subjected to static and dynamic loads commonly found in electronic devices such as sensors and electronic chips.

2. Comments and suggestions

Based on the results achieved in this thesis, several further research directions can be developed as follows:

- Studying the static and dynamic responses of micro-shell structures, taking into account the influence of geometric nonlinearity, material nonlinearity, and complex loads.

- Considering the viscoelasticity of materials and the environment in the calculation of microstructures.

- Developing numerical methods such as finite element, isometric, and meshless methods for analyzing micro-shell structures.

- Calculating the shape optimization and material optimization problems for micro-shells.

REFERENCES

- [1] J. Zhu et al., "Development trends and perspectives of future sensors and MEMS/NEMS," *Micromachines*, vol. 11, no. 1, 2020, doi: 10.3390/mi11010007.
- [2] D. C. C. Lam, F. Yang, A. C. M. Chong, J. Wang, and P. Tong, "Experiments and theory in strain gradient elasticity," *J. Mech. Phys. Solids*, vol. 51, no. 8, pp. 1477–1508, 2003, doi: 10.1016/S0022-5096(03)00053-X.
- [3] D. Van Doan, P. Van Minh, T. Van Ke, N. T. C. Nhung, and D. Van Thom, "An Overview of Functionally Graded Materials: From Civil Applications to Defense and Aerospace Industries," *J. Vib. Eng. Technol.*, vol. 13, no. 1, 2025, doi: 10.1007/s42417-024-01691-8.
- [4] Q.-H. Pham, H. T. Nhan, V. K. Tran, and A. M. Zenkour, "Hygro-thermo-mechanical vibration analysis of functionally graded porous curved nanobeams resting on elastic foundations," *Waves in Random and Complex Media*, pp. 1–32, 2023, doi: 10.1080/17455030.2023.2177500.
- [5] A. M. Dehrouyeh-Semnani and H. Mostafaei, "Vibration analysis of scale-dependent thin shallow microshells with arbitrary planform and boundary conditions," *Int. J. Eng. Sci.*, vol. 158, 2021, doi: 10.1016/j.ijengsci.2020.103413.
- [6] H. Farokhi and M. H. Ghayesh, "Nonlinear mechanical behaviour of microshells," *Int. J. Eng. Sci.*, vol. 127, pp. 127–144, 2018, doi: 10.1016/j.ijengsci.2018.02.009.
- [7] P. H. Tu, T. Van Ke, V. K. Trai, and L. Hoai, "An isogeometric analysis approach for dynamic response of doubly-curved magneto electro elastic composite shallow shell subjected to blast loading," *Def. Technol.*, vol. 41, pp. 159–180, 2024, doi: 10.1016/j.dt.2024.06.005.
- [8] H. X. Nguyen, E. Atroshchenko, H. Nguyen-Xuan, and T. P. Vo, "Geometrically nonlinear isogeometric analysis of functionally graded microplates with the modified couple stress theory," *Comput. Struct.*, vol. 193, pp. 110–127, 2017, doi: 10.1016/j.compstruc.2017.07.017.
- [9] Y. Kiani, M. Shakeri, M. R. Eslami, "Thermoelastic free vibration and dynamic behaviour of an FGM doubly curved panel via the analytical hybrid Laplace-Fourier transformation," *Acta. Mech.* 223, pp. 1199–1218 (2012), doi: 10.1007/s00707-012-0629-9.
- [10] Hoang X. Nguyen, Tuan N. Nguyen, M. Abdel-Wahab, S.P.A. Bordas, H. Nguyen-Xuan, Thuc P. Vo, "Isogeometric analysis for functionally graded microplates based on modified couple stress theory, .
- [11] H.-T. Thai, S.-E. Kim, "A size-dependent functionally graded red dy plate model based on a modified couple stress theory," *Composites Part B: Engineering* 45 (1) (2013) 1636 - 1645.

doi:<http://dx.doi.org/10.1016/j.compositesb.2012.09.065>

- [12] D. M. Tien, D. V. Thom, P. V. Minh, N. C. Tho, T. N. Doan, D. N. Mai, “The application of the nonlocal theory and various shear strain theories for bending and free vibration analysis of organic nanoplates,” *Mech. Based Des. Struct. Mach.*, vol. 52, no. 1, pp. 588-610, 2024, doi: 10.1080/15397734.2023.2186893
- [13] R. Aghababaei and J. N. Reddy, “Nonlocal third-order shear deformation plate theory with application to bending and vibration of plates,” *J. Sound Vib.*, vol. 326, no. 1-2, pp. 277-289, 2009, doi: 10.1016/j.jsv.2009.04.044.
- [14] T. V. Do, T. Q. Bui, T. T. Yu, D. T. Pham, and C. T. Nguyen, “Role of material combination and new results of mechanical behavior for FG sandwich plates in thermal environment,” *J. Comput. Sci.*, vol. 21, pp. 164-181, 2017, doi: 10.1016/j.jocs.2017.06.015.
- [15] B.L. Liu, S. Li, Y.S. Li, “Bending of FGM sandwich plates with tunable auxetic core using DQM,” *Eur. J. Mechanics / A Solids*, vol. 97, 104838, 2023, doi: 10.1016/j.euromechsol.2022.104838.